Energy transfer in two-wave mixing quasi-degenerated in photorefractive crystals

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Abstract. Wave equations describing the quasi-degenerated two-beam coupling in the photorefractive crystals for high frequency gratings have been solved. In the case of the quasi-degenerate two-beam coupling, equations depend upon the coupling coefficient, the response time of the medium, the input beams frequencies, the absorption coefficient and the input intensity ratio. The response time of the medium is function of the diffusion field, the drift field, the saturation field and the concentration ratio, i.e., the ratio between N_A density of acceptor impurities and N_D density of donor impurities. The effect of these parameters on the gain has been studied in details.

1. Introduction

Photorefractive crystals make possible an energy transfer from the reference or pump beam into the signal beam. The coupling of interacting beams in dynamic hologram recording in photorefractive media has been extensively studied due to their potential applications in signal processing, optical communications, optical computing, real-time holography, image enhancement and holographic memories. The conventional theoretical description of the waves coupling is known as the coupled wave's theory.

The nonuniform pattern received by the crystal results in a refractive index change due to the photorefractive and linear electro-optic effect. This index grating can be considered a volume dynamic grating. In experiments using non stationary conditions of recording, the energy transfer between the two beams takes place due to a phase mismatch between the incident light intensity grating in the crystal and the photoinduced index grating produced inside the crystal. In our proposal, wave equations describing the quasi-degenerated two-beam coupling in the photorefractive crystals have been analysed. The beams intensities dependence on the material response, the diffusion field, the saturation field, the drift field, the characteristic time of the medium and the concentration ratio are discussed.

2. Theoretical discussion

When two laser beams with different frequencies are incident in a photorefractive medium, a non-stationary interference fringe pattern is generated. Let us define w_1 and w_2 as the frequency of the high (reference) and low (signal) intensities beams, respectively. The electric field of each beam can be written as:

$$E_j = A_j \, e^{i(w_j t - \vec{k}_j \cdot \vec{r})} \quad j = 1,2 \tag{1}$$

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Where A_1, A_2 are the amplitudes and $\vec{k}_1, \vec{k}_2$ are the wave vectors. The resulting intensity is:

$$I = A_1 A_1^* + A_2 A_2^* + A_1 A_2^* e^{-i(\Omega t - \vec{k}_1 \cdot \vec{r})} + A_2 A_1^* e^{i(\Omega t - \vec{k}_2 \cdot \vec{r})} \tag{2}$$

Where $\Omega = w_2 - w_1$; $\vec{K} = \vec{k}_2 - \vec{k}_1$ and * indicate the complex conjugate. The intensity distribution given by Equation (2) represents a fringe pattern moving with a speed: $v = \frac{\Omega}{k} = \frac{\Omega \Lambda}{2\pi}$ where Λ is the fringe pattern period. The refractive index including the fundamental component of the intensity-induced grating can be written [1] as,

$$n = n_0 + \left[\frac{n_1}{2} e^{i\varphi} \frac{A_1^* A_2}{I_0} e^{i(\Omega t - \vec{k} \cdot \vec{r})} + c.c \right] \tag{3}$$

Where $I_0 = I_1 + I_2 = |A_1|^2 + |A_2|^2$, n_0 is the nonpertubated index of the material, ϕ indicates the shift of the photoinduced index grating with respect to the recording interference pattern and the values of ϕ is [2, 3]:

$$\phi = \phi_0 + tan^{-1}(\Omega \tau)$$

Where ϕ_0 is a constant phase shift related to the non-local response of the crystal under the interference fringe illumination and n_1 is

$$n_1 = \frac{2\Delta n_s}{\left(1 + \Omega^2 \tau^2\right)^{1/2}} \tag{4}$$

Where Δn_s is the saturation value of the photo-induced index change. Δn_s and ϕ_0 depend not only on the grating spacing and its direction but it depend on the material properties. In photorefractive medium when only the diffusion transport mechanism is consider (i.e., no external static electric field), $\varphi_0 = \frac{\pi}{2}$ [4]. On the basis of band transport model in which the materials rate equations are solved for moving grating under the assumption $I_1 << I_0$, the response time τ is given by the relation [6, 7]:

$$\tau = t_0 \left[\frac{E_d + E_\mu}{E_d + E_q} \right]; \qquad t_0 = \frac{N_A}{N_D s I_0} = \frac{1}{r s I_0}; \qquad E_d = \frac{k K_B T}{q}; \qquad E_q = \frac{q N_A}{\varepsilon_e k}; \qquad E_\mu = \frac{\gamma_r N_A}{\mu_e k} \tag{5}$$

Where t_0 is the characteristic time constant of the medium, E_d is the diffusion field, E_q is the saturation field and E_μ is the characteristic pseudo field, N_A is the density of acceptor impurities, N_D is the density of donor impurities, s is the cross-section of photo-excitation, K_B is the Boltzmann constant, T is the temperature, q is the electronic charge, γ_r is the recombination constant, ε_e is the effective dielectric constant and μ_e is the effective mobility.

The presence of the phase shift between the interference pattern and the photoinduced volume index grating allows the possibility of the non-reciprocal steady-state transfer of energy between the reference and signal beams. The coupled wave equations for the case quasi-degenerated are given by:

$$\frac{dI_1}{dz} = -\left[\frac{\gamma_0}{1 + \Omega^2 \tau^2} \right] \frac{I_1 I_2}{I_0} - I_1 \alpha \tag{7}$$

$$\frac{dI_2}{dz} = \left[\frac{\gamma_0}{1+\Omega^2\tau^2}\right]\frac{I_1 I_2}{I_0} - I_2\alpha \tag{8}$$

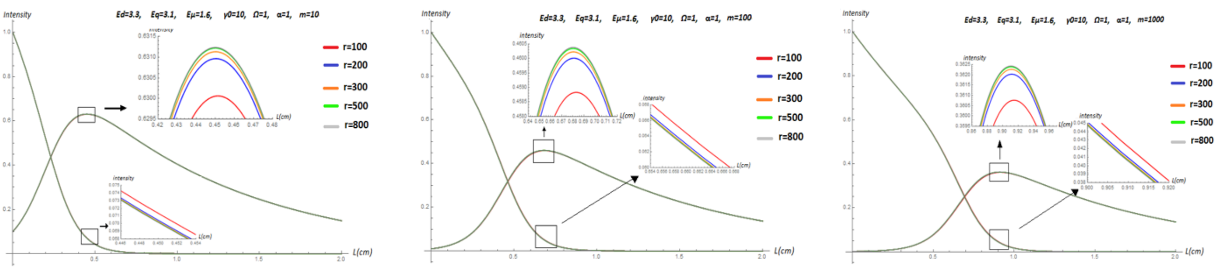
Where γ_0 is coupling constant in the degenerated case. Equations (7) and (8) can be integrated to yield:

$$I_1(z) = I_1(z)\frac{1+m^{-1}}{1+m^{-1}\exp\left[\gamma_0 L\left(1+\Omega^2\left(\frac{1}{rsI_0}\left(\frac{E_d+E_\mu}{E_d+E_q}\right)\right)^2\right)^{-1}\right]}\exp(-\alpha L)$$

$$I_2(z) = I_2(z)\frac{1+m}{1+m\exp\left[-\gamma_0 L\left(1+\Omega^2\left(\frac{1}{rsI_0}\left(\frac{E_d+E_\mu}{E_d+E_q}\right)\right)^2\right)^{-1}\right]}\exp(-\alpha L) \tag{9}$$

3. Results

Equation (9) is the beams intensities analytical expression when transfer of energy in two-wave mixing for the quasi-degenerated case is produced. It is evident that the intensity of beam I_2 increases with increasing crystal thickness reaches a maximum and then decreases exponentially due to the material absorption. Note that, for values of r higher than 500, the intensity I_2 is independent of concentration ratio r. Moreover for larger value of m, the maximum of intensity reaches at higher values of crystal thickness Figure 1(a–c).



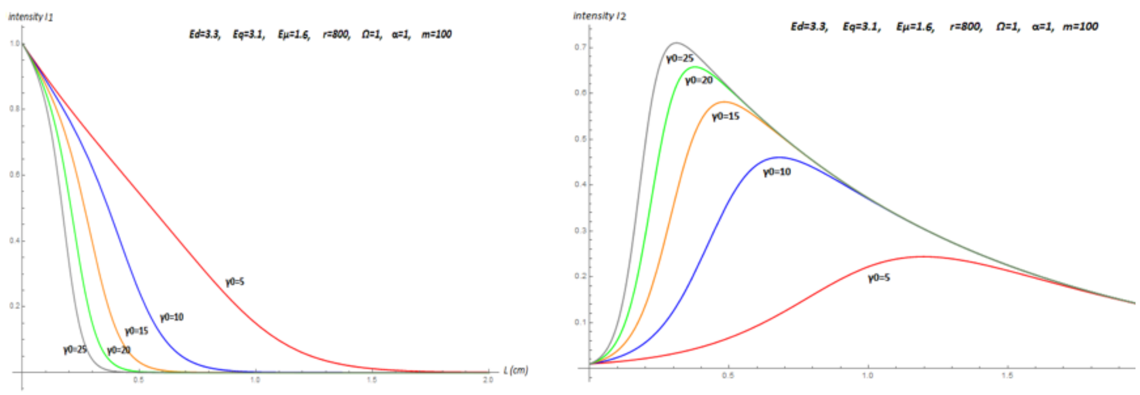
(a) (b) (c)

Figure 1. Dependence of the beams intensities on the crystal thickness and concentration ratio $r=N_D/N_A$ in quasi-degenerated two wave mixing (grating spatial frequency 2000lines/mm, Ed=3.3kV/cm; Eq=3.1kV/cm; $E\mu$=1.6kV/cm; γ_0=10; Ω=1 at various values of (a) m=10, (b) m=100, (c) m=1000.

The intensity of the beams I_1 and I_2 in term of the crystal thickness in a quasi-degenerated two wave coupling for a 2000lines/mm recording grating and different values of energy coupling coefficient γ_0=5,10,15,20,25cm^{-1} is show in Figure 2.

The results presented in Figure 3 confirm that the optimum transfer energy occurs at the same interaction length (L=0.68cm) for the different values of Ω=0.1, 0.3, 0.6, 0.8, 1.0Hz. For values of Ω lower than 0.3, the maximum intensity I_2 is independent of oscillation frequency shift.

The intensities of the reference and signal beams with respect to the concentration r for BSO and $BaTiO_3$ crystals are plotted in Figure 4(a) y 4(b) for m=100. Note that the beam I_1 decreases rapidly for lower values of r for BSO compared with $BaTiO_3$ while that the beam I_2 increases with increasing concentration ratio and intensity maximum is higher for BSO at lower values of concentration ratio than the $BaTiO_3$ material it due to the values of dielectric constant.



(a) (b)

Figure 2. Intensity of the beams in two wave coupling quasi-degenerated case, (a) I_1 (b) I_2, in terms of the crystal thickness corresponding to a 2000 lines/mm recording grating and different values of energy coupling coefficient γ_0=5, 10, 15, 20, 25cm^{-1} (Ed=3.3kV/cm; Eq=3.1kV/cm; Eμ=1.6kV/cm; m=100; r=800; Ω=1).

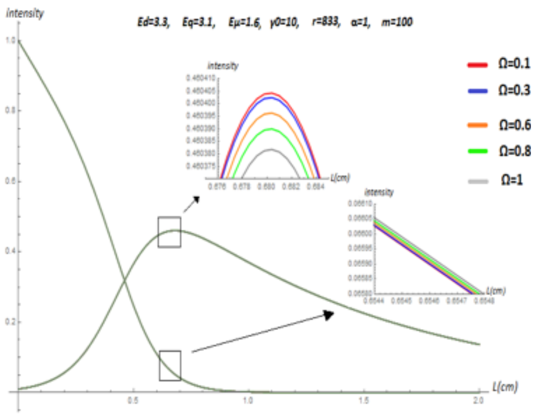
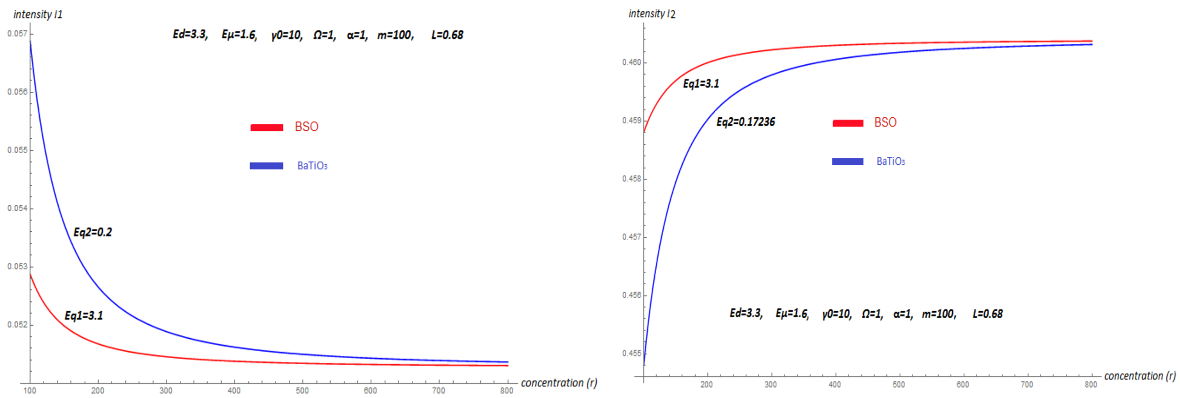


Figure 3. Dependence of the beams intensities in term of the crystal thickness for different values of Ω (Ed=3.3kV/cm; Eq=3.1kV/cm; Eμ=1.6kV/cm; m=100; γ_0=10; r=833).



(a) (b)

Figure 4. Intensities of the reference and signal beams (a) I_1 and (b) I_2 with respect to the concentration ratio r=N_D/N_A for BSO and $BaTiO_3$ materials in two beam coupling quasi-degenerated case (2000 lines/mm recording grating, Ed=3.3kV/cm; Eμ=1.6 kV/cm; m=100; γ_0=10; Ω=1; L=0.68).

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4. Conclusions

It is clear that the intensity of the two beams (reference and signal) inside of the crystal thickness not only depend on the coupling coefficient γ_0, the crystal thickness L, the modulation ratio m, the absorption coefficient and the oscillation frequency shift Ω. Also, the mentioned intensities change with the density of acceptor impurities N_A and the density of donor impurities N_D ($r=N_D/N_A$). The transfer energy between the two beams increases with crystal thickness. The maximum intensity I_2, increase when the concentration ratio increases. Indeed, it is observed higher values for BSO crystal at lower values of concentration ratio than $BaTiO_3$ material.

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